

Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Decision Mathematics 1 (6689/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

- 1. The total number of marks for the paper is 75
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method
 (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- d... or dep dependent
- indep independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper or ag- answer given
- _ or d... The second mark is dependent on gaining the first mark

- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
- 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1.(a)	AG, DG, AF; AE BG; CD	M1 A1 A1 (3)
(b)	F G B C D	B1 (1)
(c)	Weight of tree = 298 (s)	B1 (1) 5 marks

a1M1: First three arcs correctly chosen in order (AG, DG, AF, ... or weights 42, 41, 48, ...) or first four nodes correctly chosen in order. {A.G,D,F,...} If any rejections seen at any point then M1 (max) only. Order of nodes may be seen at the top of the matrix {1, -, -, 3, -, 4, 2} so please check the top of the matrix carefully.

a1A1: First five arcs correctly chosen in order (AG, DG, AF, AE, BG or weights 42, 41, 48, 50, 58,...) or all seven nodes correctly chosen in order. {A,G,D,F,E,B,C}. Order of nodes may be seen at the top of the matrix so for the first two marks accept {1, 6, 7, 3, 5, 4, 2} (do not condone any missing numbers e.g. the number 7 must be above C).

a2A1: CSO - all **arcs** correct and chosen in the correct order. They must be considering arcs for this final mark (do not accept a list of the weights of each arc, nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen).

Misread: Starting at a node other than A scores **M1 only** – **must** have the first three arcs (or four nodes or numbers) correct (and in the correct order).

Starting at	Minimum arcs	Nodes	Order
	required for M1		
A	AG DG AF	AGDF(EBC)	1(6)(7)3(5)42
В	BG DG AG	BGDA(FEC)	41(7)3(6)(5)2
С	CD DG AG	CDGA(FEB)	4(7)12(6)(5)3
D	DG AG AF	DGAF(EBC)	3(6)(7)1(5)42
Е	EA AG DG	EAGD(FBC)	2(6)(7)41(5)3
F	FA AG DG	FAGD(EBC)	2(6)(7)4(5)13
G	GD AG AF	GDAF(EBC)	3(6)(7)2(5)41

b1B1: CAO (condone lack of weights on arcs).

c1B1: CAO (condone lack of units).

Question Number	Scheme	Mark	s
2. (a)	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	M1 A1 A1 A1 A1	(5)
(b)	1 st dummy – A and B both must be able to be described uniquely in terms of the events at each end. 2 nd dummy – I depends on D only but J depends on D and G.	B1 B1 7 marks	(2)

Throughout part (a) condone lack of numbered events throughout – also 'dealt with correctly' means that the activity starts from the correct event (but not necessarily finishing at the correct event) e.g. 'H dealt with correctly' requires F and E leading into the same event and H starting from that event (but not necessarily H leading into K). Activity on node is M0.

a1M1: 7 activities and one dummy placed.

a1A1: One start + activities A, B, C and E dealt with correctly.

a2A1: Activities D, F, G, H and K and the 1st dummy dealt with correctly.

a3A1: Activities I and J and the 2nd dummy dealt with correctly.

a4A1: CSO (all four previous marks must have been awarded) - **all** arrows present and correctly placed with one finish – condone lack of arrows for the first four marks only. No 'extra' activities.

Note that another valid solution would be the dummy going from event 3 to event 2 and D, G and F coming out of event 2. Or the candidate could start with a dummy from event 1 to ensure the uniqueness of activities A and B.

b1B1: CAO – with no incorrect terminology (e.g. event for activity) - please note that e.g. 'so that activities can be defined uniquely' is not sufficient to earn this mark. There must be a mention of describing activities uniquely **in terms of the events at each end**. However give bod on statements that imply that an activity begins and ends at the same event e.g. 'so that activites do not have the same start and finish' is sufficient for B1.

b2B1: CAO – all relevant activities must be referred to – so activities D, G, I and J must all be mentioned for this mark.

Question Number	Scheme	Marks	
	D(A)E + F(J)K = 35 + 15 = 50*	M1	
	D(HJ)F + E(FJ)K = 24 + 40 = 64	A1 (2 correc	et)
3. (a)	D(HJ)K + EF = 33 + 25 = 58	A1 (3 correc	et)
	Arcs DA, AE, FJ, JK will be traversed twice	A1	
	Route length = $451 + 50 = 501$ (km)	A1ft	(5)
(b)	Vertex J would appear 3 times in the shortest inspection route	B1	(1)
	We only have to repeat one pair of odd vertices which does not include vertex K (DE = 35, DF = 24, EF = 25)	M1	
(c)	DF is the smallest of the three so repeat DF (DH, HJ, JF) and therefore the other hut should be built at E	A1	
(c)	Route e.g. EADEHDHJFBEFCGFJHLGKJLMK	A1	
	The length of the route is 475 (km)	A1ft	(4)
		10 marks	

- a1M1: Three distinct pairings of the **correct** four odd nodes.
- a1A1: Any two rows correct including pairings and totals.
- a2A1: All three rows correct including pairings and totals.
- a3A1: CAO correct **arcs** clearly (**not** just in their working) stated: DA, AE, FJ, JK. Accept DAE, FJK **or** DE via A, FK via J. Do not accept DE, FK.
- a4A1ft: The correct answer of 501 or 451 + their smallest repeat out of a choice of at least **two** totals seen.

b1B1: CAO (3)

- c1M1: Identifies the need to repeat one pairing not including K (maybe implicit) or listing of possible repeats this mark is dependent on scoring the M mark in (a). Stating any pairing that does not include K is sufficient for this mark.
- c1A1: Identifies DF as the least of those pairings not including K and E as the position of the other hut. They have to explicitly state that DF is the least pairing that does not include K or they can list all three pairings (DE, DF, EF) and then say DF is the smallest as this implicitly implies that they are considering only pairings that do not include K.
- c2A1: Any correct route (checks: starts at E and finishes at K (or vice-versa), 24 vertices (D, G, L appear twice and E, F, H, J appear three times and every other letter appears at least once).
- c3A1ft: Correct answer of 475 or 451+ their DF (i.e. the least pairing that does not include K so their smallest of DE, DF or EF).

Question Number	Scheme	Marks
4. (a) (b)	C A C T T T V J V N D P D D G G R G	B1 (2)
(c)	Alternating path either $N-V=F-T=A-C=J-D$ or $P-V=F-T=A-C=J-D$ Change status $N=V-F=T-A=C-J=D$ or $P=V-F=T-A=C-J=D$ Improved matching $A=C, F=T, J=D, N=V, (P \text{ unmatched}), R=G$	M1 A1
(d)	or $A = C$, $F = T$, $J = D$, (N unmatched), $P = V$, $R = G$ e.g. both K and G can only be allocated to R	A1 (3) B1 (1)
	e.g. N and P can only be allocated to V	M1
(e)	Change status $P = D - J = C - A = T - F = G - R = K$ or $N = V - P = D - J = C - A = T - F = G - R = K$	A1
	Complete matching $A = T$, $F = G$, $J = C$, $N = V$, $P = D$, $R = K$	A1 (3) 9 marks

a1B1: CAO – condone the addition of an arc from F to G and/or one from P to D only.

b1B1: CAO – these four arcs and no additional ones.

c1M1: An alternating path (e.g. letter 1^{st} set – letter 2^{nd} set – letter 1^{st} set - ...) from **either** N **or** P **to** D – or vice versa.

c1A1: CAO – a correct path including change status **either** stated (only accept 'change (of) status' **or** 'c.s.') **or** shown (**all** symbols e.g. (...-...=...) **interchanged** (...=...-...=...). Chosen path clear.

e.g.

•
$$N - V = F - T = A - C = J - D$$

 $N = V - F = T - A = C - J = D$

Scores M1A1 (change status shown)

• change status
$$N - V = F - T = A - C = J - D$$

Scores M1A1 (change status stated)

• c.s.
$$N - V = F - T = A - C = J - D$$

Scores M1A1 (change status stated)

•
$$N-V = F-T = A-C = J-D$$

c.s. $N = V-F = T-A = C-J = D$

Scores M1A1 (change status stated and shown)

•
$$N - V = F - T = A - C = J - D$$

 $N = V, F = T, A = C, J = D, ...$

Scores M1A0 (no change status stated or shown)

c2A1: CAO must follow from the correct stated path. Accept on a **clear** diagram (with five arcs **only**). Condone lack of P or N being stated as unmatched.

d1B1: CAO (completely correct statement) – do not accept a general statement (specific nodes must be referred to). Note that these need to be checked carefully e.g. V can only be allocated to N and P is B0.

e1M1: A second alternating path from either N (if P used in (c)) or P (if N used in (c)) to K (or vice-versa) e1A1: CAO including change status (stated **or** shown), chosen path clear.

e2A1: CAO must follow from **two correct** stated paths (so **both** previous M marks must have been awarded). Accept on a **clear** diagram (with six arcs only).

Question Number	Scheme	Marks
5. (a)	B 2 10 34 S 8 42 44 42 48 10 6 26 26 26 90 89 90 89 90 89 17 12 18 17 14 42 42 48 17 14 42 48 17 18 17 18 17 18 17 18 17 18 17 18 18	M1 A1 (PBCAW) A1 (HMS) A1ft (LY)
	Shortest route: $P - B - A - S - L - Y$ Length: 89 (miles)	B1 B1ft (6)
(b)	Shortest route: $P-C-H-M-L-Y$ Difference in routes: $(41 + 40 + 21) - 89 = 13$ (miles)	B1 M1 A1 (3) 9 marks

In part (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at L the working values must be 70 69 68 – in that order (70 68 69 is incorrect).

The values in brackets in the working values at P, A, H and L can be ignored but if a candidate does have additional values at these nodes then they must be these ones only. Penalise any other/incorrect working values with the corresponding A mark. It is also important that the order of labelling is checked carefully – some candidates start with a working value of 0 at P (rather than 1) – this is fine. Also the order of labelling must be a strictly increasing sequence – so $1, 2, 3, 3, 4, \ldots$ will be penalised once (see notes below) but $1, 2, 3, 5, 6, \ldots$ is fine.

a1M1: A larger value replaced by smaller value at least once in the working values at either A or M or L or S or Y.

a1A1: All values in P, B, C, A and W correct. The working values at A must be in the correct order. Condone lack of 0 in P's working value. Ignore additional working value of 30 at the end of A (may read 20 16 30 – rather than 20 16 - at A).

a2A1: All values in H, M and S correct and the working values in the correct order. Penalise order of labelling only once per question (H, M and S labelled in that order and H must be labelled after P, B, C, A and W). Ignore additional working value of 33 at the end of H (may read 27 33).

a3A1ft: All values in L and Y correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question (L and Y labelled in that order and L labelled after all other nodes). Ignore additional working value of 81 at L - may read 70 69 81 68 – rather than 70 69 68 – which is fine – however, 70 69 68 81 is incorrect and loses this mark.

To follow through check that all the working values at L follow from the candidate's final values from nodes A, H, M and S in whatever order the candidate has labelled these four nodes and that the final value and order of labelling follows through correctly. Repeat for Y (which will have working values from S and L).

a1B1: CAO for the route

a2B1ft: Follow through on their final value at Y – if answer is not 89 ft their final value at Y (condone lack of units)

b1B1: CAO for the route

b1M1: Their final value at M + 40 + 21 - accept a value of 102 (with no other working) for this mark. b1A1: CAO (condone lack of units) – accept, as a minimum,102 followed by 13 for both marks. If 13 with no working then award the previous M mark but withhold the final A mark

Question Number			Marks								
6. (a)	Bin Bin	1: 24 2: <i>x</i> 3: 25 4: 9	M1 A1 A1 (3)								
(b)	e.g. 24 24 24 25 25 25	using 1 14 25 24 24 24	middle 8 25 <u>x</u> <u>x</u> <u>x</u> <u>x</u> <u>x</u>	e right x 19 19 19 19 19 19	19 14 14 17 17 17	25 8 8 14 14 14	6 6 17 8 9 9	17 17 9 8 8	9 9 <u>6</u> <u>6</u> <u>6</u>	pivot 19 pivots x 6 pivots 25 17 pivots (24) 8 pivot 9 (sort complete)	M1 (quick sort) A1 (1 st pass/pivots for 2 nd) A1ft (2 nd and 3 rd passes correct) A1cso (4)
(c)	В	Bin 1: 2 in 2: . in 3: 1	x 19	9		(ii)		: x 19			M1 A1 A1 A1 (4)
(d)	<i>x</i> +	19 + 9 19 + 8	$\theta = 50$	$\Rightarrow x$							B2,1,0 (2) 13 marks

a1M1: First four items placed correctly and at least six values put in bins (so bin 1 correct and the x in bin

2). If a candidate gives x a value in the given interval then allow this for the M mark in (a) only.

a1A1: First seven items placed correctly (so bins 1 and 2 correct and 25 in bin 3)

a2A1: cso – all correct

b1M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as pivot is M0) and first pass gives >p, p, <p. So after the first pass the list should read (values greater than the pivot), pivot, (values less than the pivot). **If only choosing one pivot per iteration M1 only**

b1A1: First pass correct, next two pivots chosen correctly for second pass. If a candidate gives *x* a value in the given interval then allow this for the M mark and first A mark only in (b).

b2A1ft: Second and third passes correct (follow through from their first pass and choice of pivots) – need not be choosing pivot(s) for the fourth pass for this mark.

b3A1: CSO (correct solution only- all previous marks in this part **must** have been awarded) including choice of pivots for the fifth pass and 'sort complete' – this could be shown **either** by a 'stop' statement **or** final list being re-written **or** using each item as a pivot.

c1M1: **Must be using 'sorted' list** in decreasing order. First four items placed correctly and at least six values put in bins (so bin 1 correct and the *x* and 19 in bin 2). If a candidate has given *x* a value in (c) then M0.

c1A1: First six values correct (bin 1 correct, the x and 19 in bin 2, the 17 and 14 in bin 3)

c2A1: One allocation correct

c3A1: Both allocations correct - both allocations must be clear.

d1B1: A correct value of x stated (working not necessary) – dependent on one correct allocation in (c).

d2B1: Both values correctly calculated (with relevant working) – dependent on both allocations correct seen

in (c). If more than two values for x stated (e.g. all possible integer values) then no marks in (d).

SC for (c): if 'sorted' list has one error from (b) (e.g. a missing number, an extra number or one number incorrectly placed) then M1A1 can be awarded in (c) (for four items (M1) and six items (A1) correctly placed - see above). However no marks in (d). If there is more than one error then M0.

Part (b) Using middle left as pivot

24	14	8	\boldsymbol{x}	19	25	6	17	9	pivot 19	
<u>24</u>	\boldsymbol{x}	25	<u> 19</u>	14	8	6	17	9	pivots x 6	M1 A1
24	25	<u>X</u>	<u> 19</u>	14	8	1 7	9	<u>6</u>	pivots 24 8	
25	<u>24</u>	<u>x</u>	<u> 19</u>	14	17	<u>8</u>	9	<u>6</u>	pivots (25) 14 (9)	A1
25	<u>24</u>	<u>x</u>	<u> 19</u>	17	<u>14</u>	9	<u>8</u>	<u>6</u>	sort complete	A1cso

Misreads

• If they have used the correct numbers at any point in part (a) and then use incorrect numbers in part (b) (say 71 instead of 17) from the beginning of the sort or misread their own numbers **during part** (b) then count it as an **error in part** (b) (so they will lose at least the final A mark but should be able to gain at least the M mark and ft A mark) – then mark part (c) according to the SC above.

Sorting list into ascending order in (b)

- If the candidate sorts the list into ascending order and reverse the list **in part** (**b**) then they can score full marks in (b).
- If the list is not reversed in part (b) then mark as a misread (so remove the last two A marks if earned in (b)). If the list is reversed at the start of (c) but not in (b) then still treat this as a misread. If the list is still in ascending order in part (c) award no marks for first fit increasing. If the candidate says that the list needs reversing in part (b) but doesn't actually show the reversed list in (b) then remove the final A mark in (b).

Ascending (middle left)									As	cendi	ng (ı	middl	e righ	t)				
2		8	x	19	25	6	17	9	M1	24	14	8	x	19	25	6	17	9
1		6	17	9	<u>19</u>	24	x	25	A1	14	8	6	17	9	<u>19</u>	24	x	25
6		8	17	9	<u>19</u>	\underline{x}	24	25		<u>6</u>	14	8	17	9	<u>19</u>	\underline{x}	24	25
9		14	<u>17</u>	9	<u>19</u>	<u>X</u>	<u>24</u>	25	A1	<u>6</u>	14	8	9	<u>17</u>	<u>19</u>	<u>x</u>	24	<u>25</u>
9	<u>8</u>	14	9	<u>17</u>	<u>19</u>	\underline{x}	<u>24</u>	25		<u>6</u>	<u>8</u>	14	9	<u>17</u>	<u>19</u>	\underline{x}	24	<u>25</u>
(<u>8</u>	9	<u>14</u>	<u>17</u>	<u> 19</u>	<u>x</u>	<u>24</u>	25	A1cso	<u>6</u>	<u>8</u>	<u>9</u>	14	<u>17</u>	<u> 19</u>	<u>x</u>	24	<u>25</u>

Question Number	Scheme	Marks	
7. (a)	The total float $F(i,j)$ of activity (i,j) is defined to be $F(i,j) = l_j - e_i - duration (i,j), where e_i is the earliest time for event i and l_j is the latest time for event j (see note below)$	B2,1,0	(2)
(b)	E (6) F (2) A (4) C (7) H (3) $\frac{15}{17}$ $\frac{1}{17}$ B (5) $\frac{1}{11}$	M1 A1 A1	(3)
(c)	Critical activities: A C J M	B1	(1)
(d)	G can be delayed by $21 - 11 - 3 = 7$ (days)	M1 A1	(2)
(e)	$\frac{69}{30}$ = 2.3 so lower bound is 3 workers	M1 A1	(2)
(f) e.g.	0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34	M1 A1 A1 A1 A1 14 marks	(4)

a1B1: For the first mark: the idea that total float is 'how long an activity can be delayed for'. Give bod. a2B1: For both marks: A clear correct statement e.g. the **total** amount of time that an activity may be delayed from its **early** start without delaying the project **finish** time. The candidate must clearly demonstrate a knowledge that total float = latest finish – earliest start – duration of activity. Ignore comments that infer that total refers to the sum of the floats for all activities in an activity network. Note that B1B0 should be awarded for an answer that has the pertinent idea of 'float' (see a1B1 above) and B1B1

for a clear correct statement (see a2B1 above) – B0B1 cannot be awarded in this part.

b1M1: All top boxes and all bottom boxes completed. Values generally increasing from left to right (for top boxes) and values generally decreasing from right to left (for bottom boxes). Condone missing 0 or 30 for M only (for bottom boxes). Condone one rogue value in top boxes **and** one rouge value in bottom boxes (if values do not increase from left to right (or decrease right to left) then if **one** value is ignored and then the values do increase from left to right (or decrease right to left) then this is considered to be one rogue value).

b1A1: CAO for top boxes.

b2A1: CAO for bottom boxes.

c1B1: CAO

d1M1: Correct calculation for their activity G seen - all three numbers correct (ft). Final value must be non-negative.

d1A1: CAO (no follow through on this A mark). Answer of 7 with no working scores no marks in this part.

e1M1: Attempt to find lower bound [59 – 79 / their finish time]

e1A1: CAO – correct calculation seen then 3. [As 30/13 also gives 3, an answer of 3 with no working scores M0A0.]

f1M1: Not a cascade chart. 4 'workers' used at most and at least 8 activities placed.

f1A1: The critical (A, C, J, M) activities and B and D correct A - 4, C - 7, J - 10, M - 9, B - 5, D - 9. B must be completed by its late finish time (11) and D must start after A and finishing before its late finish time (15).

Now check the last 7 activities – the last two marks are for E, F, G, H, I, K and L only

First check that there are only three workers and that all 13 activities are present (just once).

Then check precedences (see table below) – each row of the table could give rise to 1 error only in precedences

Finally check the length of each activity and the time interval in which the activity must take place (interval is inclusive).

Activity	Duration	Time interval	IPA
Е	6	4 - 17	A
F	2	13 – 17	D
G	3	11 – 21	B, C
Н	3	13 – 21	D
I	4	15 – 21	E, F
K	5	14 – 30	G
L	2	14 – 30	G

f2A1: 3 workers. All 13 activities present (just once). Condone one error either precedence, time interval or activity length, on activities E, F, G, H, I, K and L only.

f3A1: 3 workers. All 13 activities present (just once). No errors on activities E, F, G, H, I, K and L.

Question Number	Scheme	Marks	
8. (a)	$y \le 2x$, $5y \ge 2x$, $2x + y \le 36$, $4x + y \ge 36$	B2,1,0	(2)
	B(6,12), C(9,18), D(15,6)	B1	
	$A\left(\frac{90}{11},\frac{36}{11}\right),$	B1	
	at A: $F = \frac{90}{11} + \frac{36}{11}k$, at B: $F = 6 + 12k$, at C: $F = 9 + 18k$, at D: $F = 15 + 6k$	B1	
(b)	$\frac{90}{11} + \frac{36}{11}k < 6 + 12k \text{ and } 9 + 18k < 15 + 6k$	M1	
	$k > \frac{1}{4}$ and $k < \frac{1}{2}$	A1	
	$\frac{1}{4} < k < \frac{1}{2}$	A1	(6)
	4 2	8 marks	

a1B1: Any two correct inequalities (condone strict inequalities).

a2B1: CAO (equalities cannot be strict for this mark).

As there are a number of different methods that the candidates can adopt – consider the candidate's full response and mark each attempt according to the notes below - award the candidate the marks for their best response/attempt. However, do not mix the approaches together e.g. if they find the exact coordinates of all four vertices and then state that the maximum gradient of P is -2 then this would score the first two marks only (method 1).

Method 1 (point testing)

b1B1: The coordinates of B, C and D stated exactly.

b2B1: The coordinates of A stated exactly.

b3B1: The objective function calculated in terms of k at either A or B or C or D.

b1M1: Either (their objective function at A) < (their objective function at B) or (their objective function at C) < (their objective function at D) (condone equals sign or any inequality).

b1A1: Either $k > \frac{1}{4}$ or $k < \frac{1}{2}$ or $k \ge \frac{1}{4}$ or $k \le \frac{1}{2}$. b2A1: CAO $\frac{1}{4} < k < \frac{1}{2}$ or $\frac{1}{4} \le k \le \frac{1}{2}$ (or as separate inequalities)

Method 2 (objective line method I)

Comparing the gradient of the objective function to the gradient of the two lines with negative gradient.

b1B1: The **minimum** gradient (of P) stated as $-4 - \mathbf{must}$ see explicit mention of minimum.

b2B1: The **maximum** gradient (of P) stated as $-2 - \mathbf{must}$ see explicit mention of maximum.

b3B1: Gradient of objective function stated as $-\frac{1}{k}$.

b1M1: Comparing gradient of objective function to either -2 or -4.

Final two marks as in method 1.

Method 3 (objective line method II)

b1B1: **Minimum** P parallel to $4x + y = \cdots$ (limiting case) – **must** see explicit mention of minimum.

b2B1: **Maximum** P parallel to $2x + y = \cdots$ (limiting case) – **must** see explicit mention of maximum.

b3B1: Re-arranging equations (either seen or implied) to give $x + \frac{y}{4} = \cdots$, $x + \frac{y}{2} = \cdots$

b1M1: Compare coefficients of y in the objective function & lines.

Final two marks as in method 1.

SC: If **no** working seen (max 3/6 marks)

 $k = \frac{1}{2}$ or $k = \frac{1}{4}$ (where \blacksquare is any inequality or equals) award first B mark. $k > \frac{1}{4}$ or $k < \frac{1}{2}$ or $k \ge \frac{1}{4}$ or $k \le \frac{1}{2}$ award the first two B marks.

 $\frac{1}{4} < k < \frac{1}{2}$ or $\frac{1}{4} \le k \le \frac{1}{2}$ award the first three B marks.